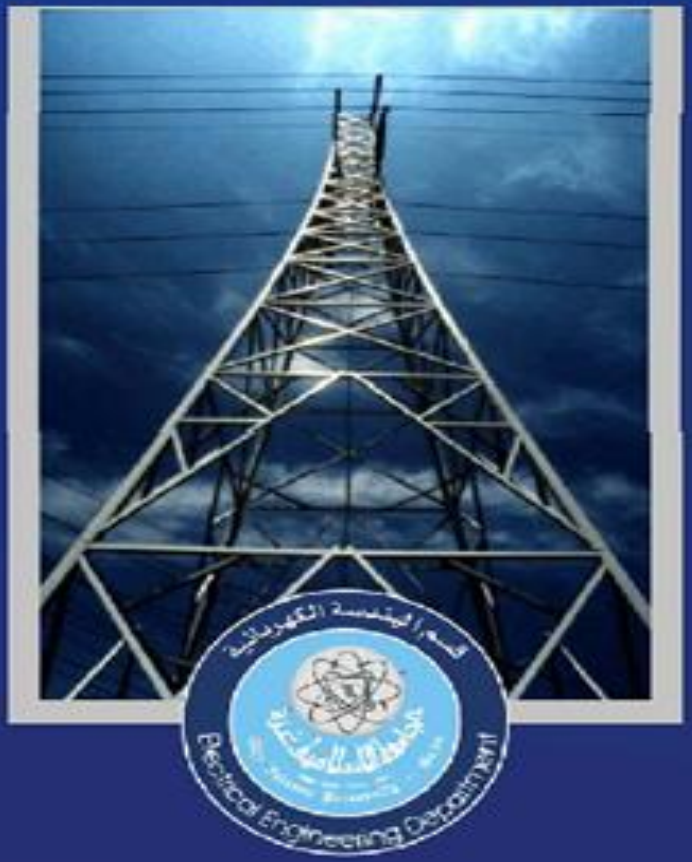
THE ISLAMIC UNIVERSITY OF GAZA



ELECTRICAL DEPARTMENT

Engineering Faculty

بسم الله الرحمن الرحيم

PROBABILITY DISCUSSION

Chapter 1

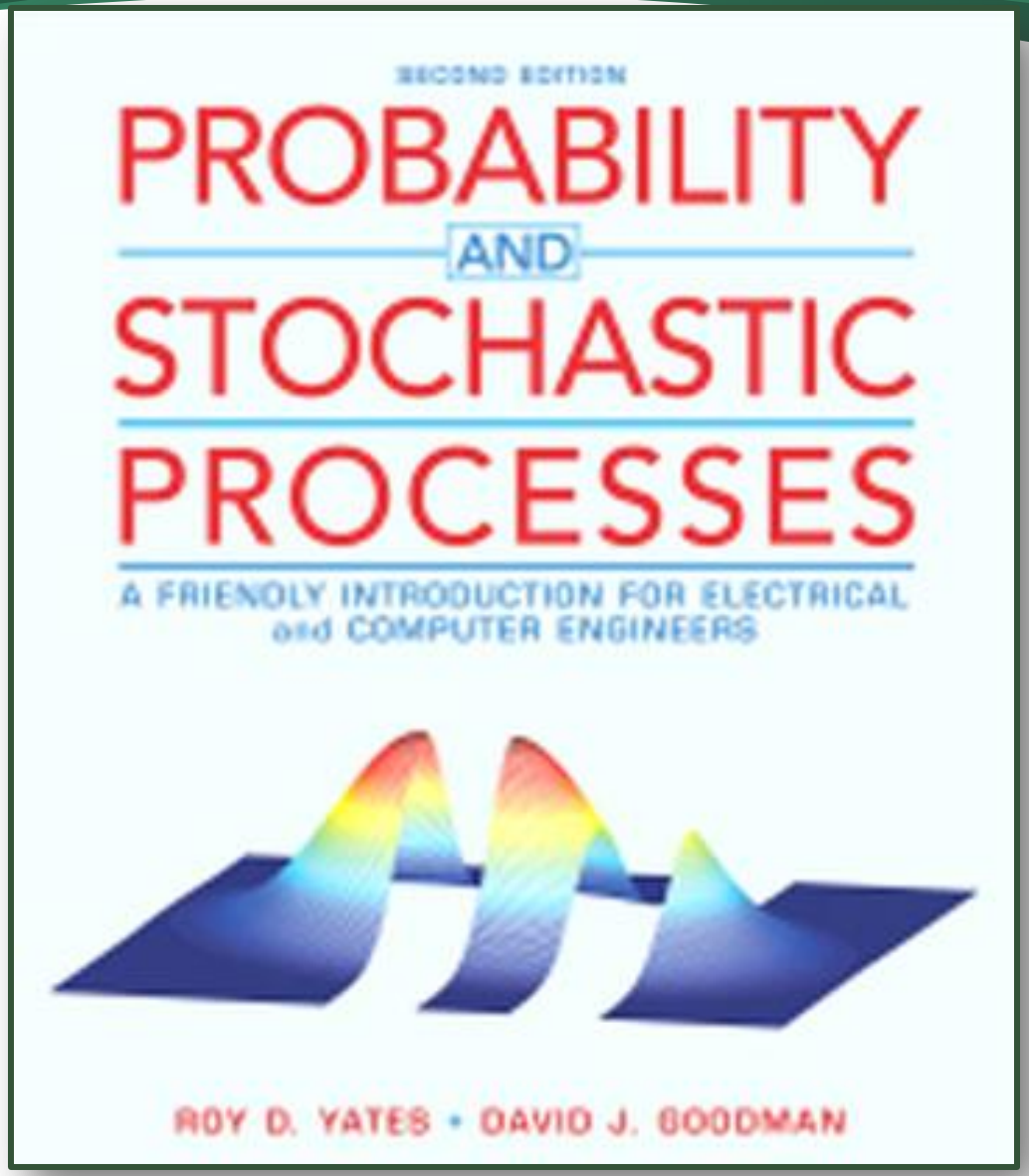
Getting Stared with Probability 'PART 1'



iugaza2010.blogspot.com
melasmer@gmail.com

Eng.Mohammed Elasmer

Fall-2011


SECOND EDITION
PROBABILITY
AND
STOCHASTIC
PROCESSES
A FRIENDLY INTRODUCTION FOR ELECTRICAL and COMPUTER ENGINEERS
ROY D. YATES • DAVID J. GOODMAN



1.1 Set Theory

- **Set** : a collection of things
- **Elements**: are things that make up the set
- Set A contain elements x,y,z

1.1 Set Theory

Some definitions:

Inclusion: $x \in A \rightarrow$ x is an element in set A

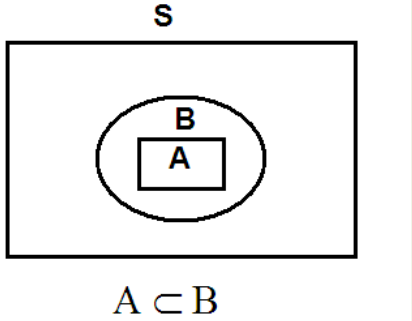
Subset: $A \subset B$

Equality: $A = B \rightarrow \ A \subset B \ \& \ B \subset A$

Universal Set (S)

Null Set (ϕ) has no elements

Ven diagram: used to display realtionships among sets, the lage rectangle is the universal set S

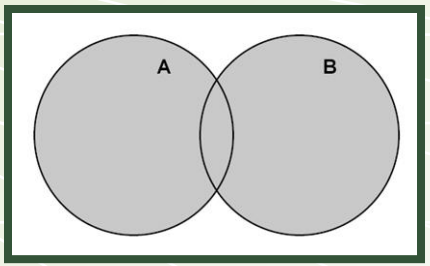


1.1 Set Theory

❑Operations in Set Algebra

(1) Union

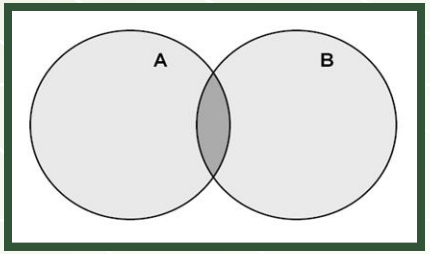
$$x \in A \cup B \text{ iff } x \in A \text{ or } x \in B$$



A or B

(2) Intersection

$$x \in A \cap B \text{ iff } x \in A \text{ and } x \in B$$

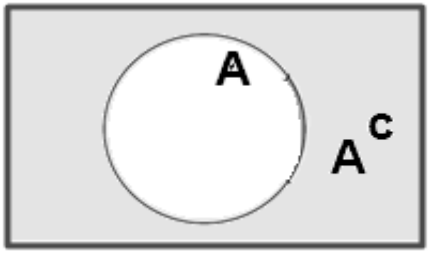


A and B

(3) Complement

$$x \in A^c \text{ iff } x \notin A$$

$$A^c = S - A$$



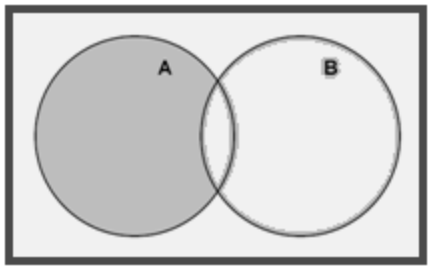
1.1 Set Theory

❑Operations in Set Algebra

(4) Difference

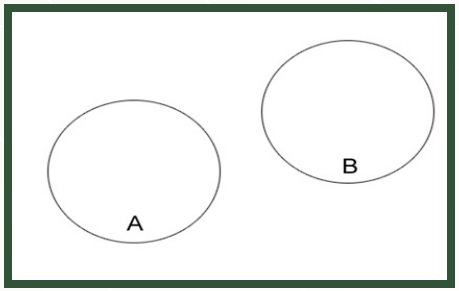
$$x \in A - B \text{ iff } x \in A \,\&\, x \notin B$$

$$A - B = A \cap B^c$$



disjoint

$$A \text{ and } B \text{ called disjoint } iff \quad A \cap B = \Phi$$

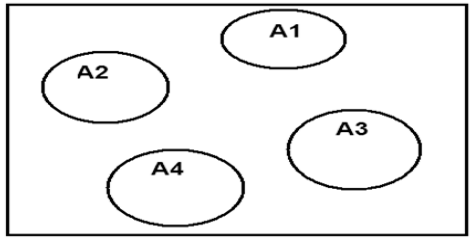


1.1 Set Theory

Mutually Exclusive متنافية

A collection of sets A1,....., An is mutually exclusive iff

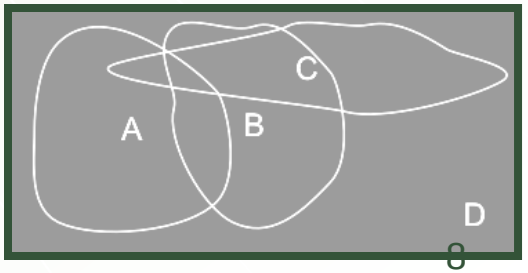
$$A_i \cap A_j = \Phi$$



Collectively Exhaustive مجتمعة شاملة

A collection of sets A1,..., An is collectively exhaustive iff

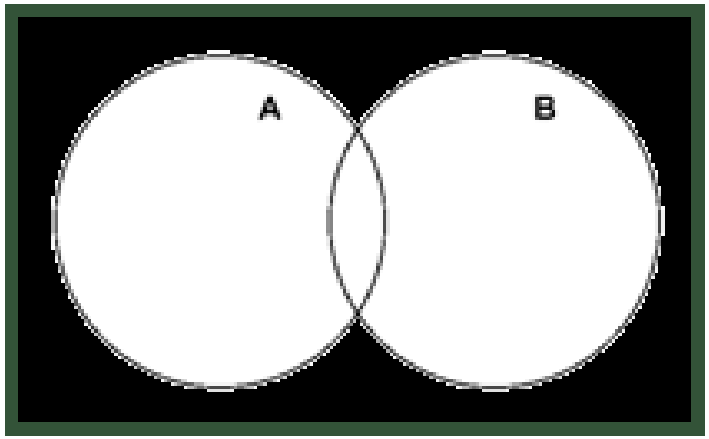
$$A_1 \cup A_2 \cup A_3 \ldots\ldots = S$$



1.1 Set Theory

DEMORGAN LAW

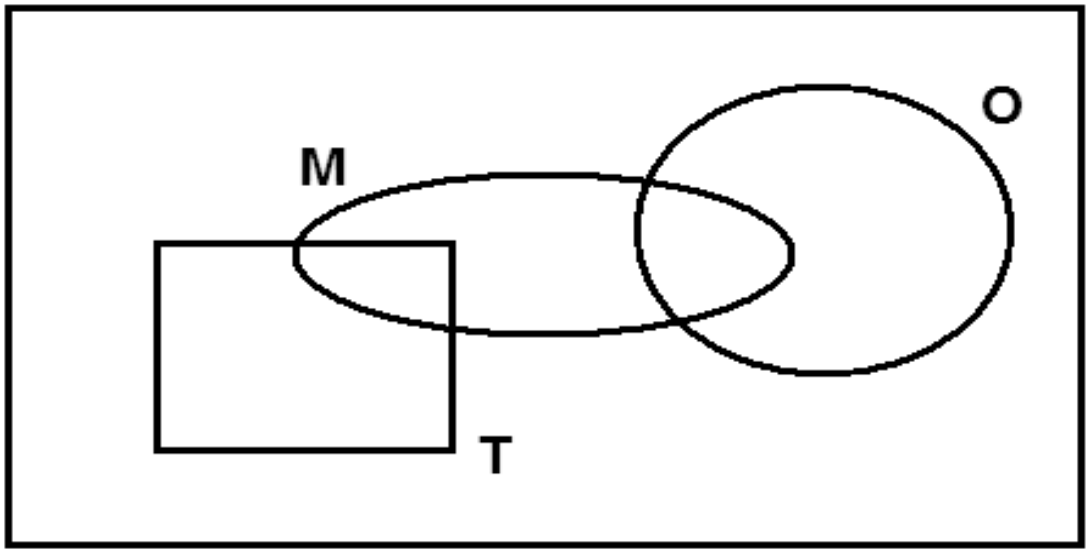
$$(A \cup B)^C = A^C \cap B^C$$

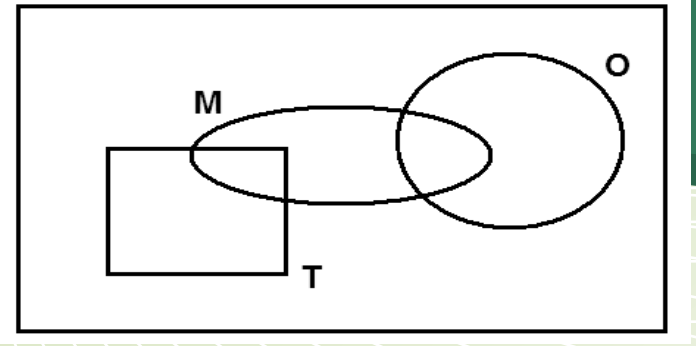


Quiz 1.1

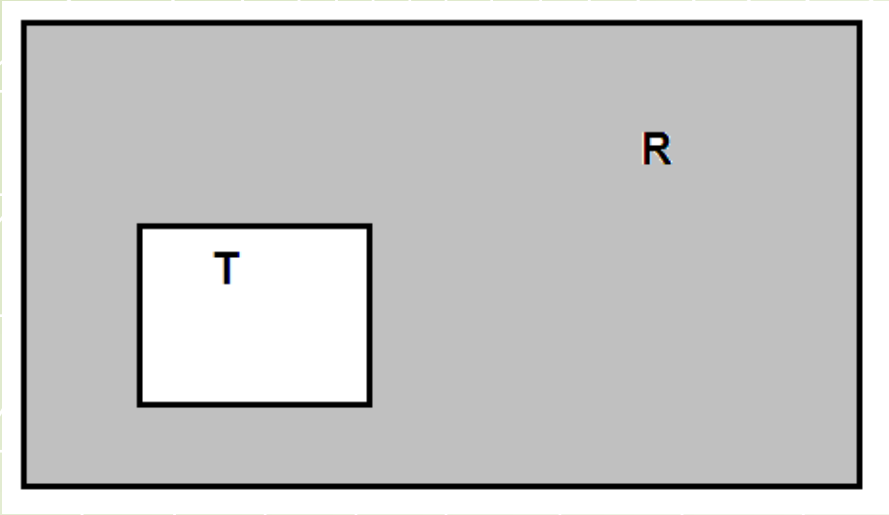
A slice of pizza sold by Gerlanda's Pizza is either
Regular Pizza(R) or Tuscan(T)
Each slice may have mushrooms(M) or onions(O)
As described by the Venn diagram.

Shade the region of Venn diagram for the following sets:

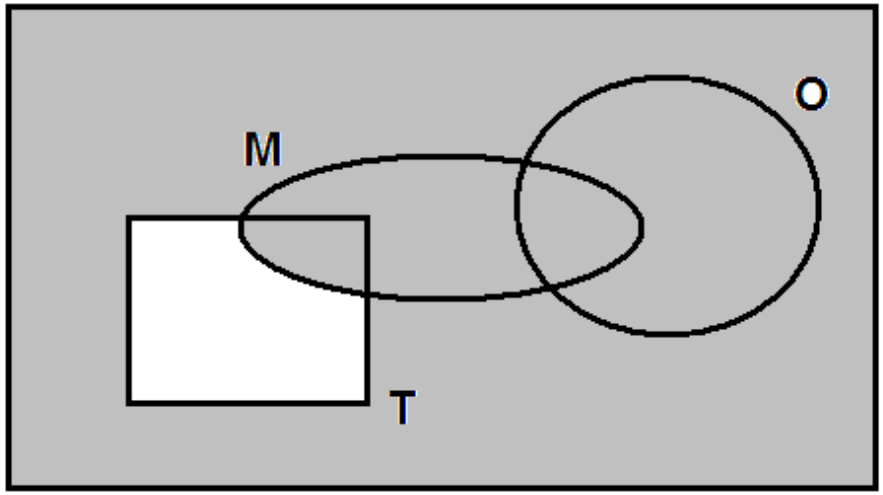




(a) R

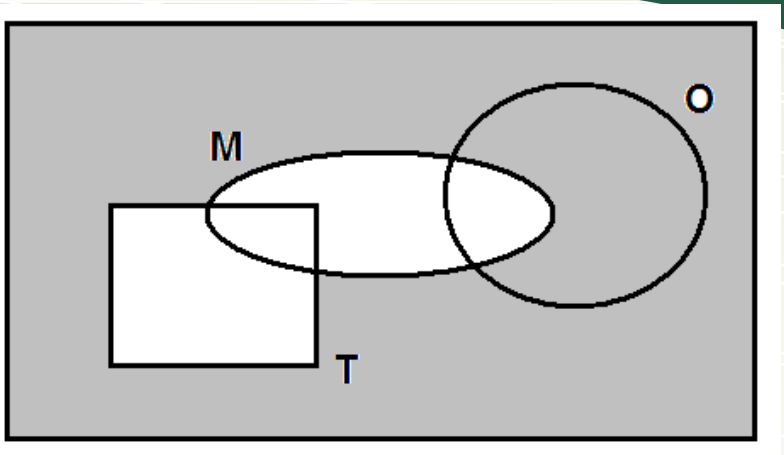


(d) R ∪ M

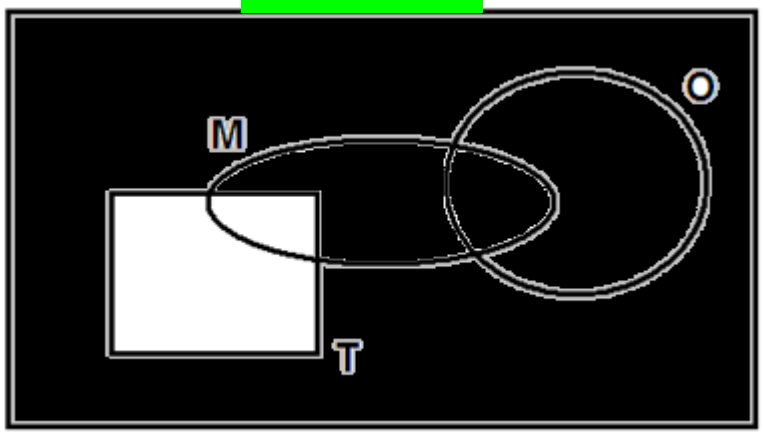


(f) $T^c - M$

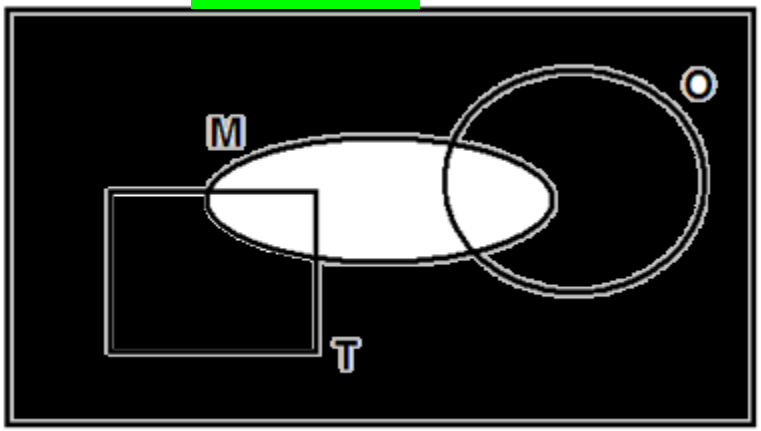
$$= T^c \cap M^c$$



T^c



M^c

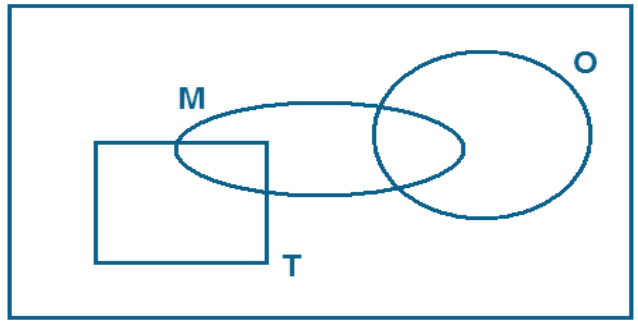


PROBLEM 1.1.1

From Quiz1.1 ask the following questions:

(a) Are T and M mutually exclusive?

$$No, T \cap M \neq \Phi$$



(b) Are R,T and M collectively exhaustive ?

$$Yes, R \cup T \cup M = S$$

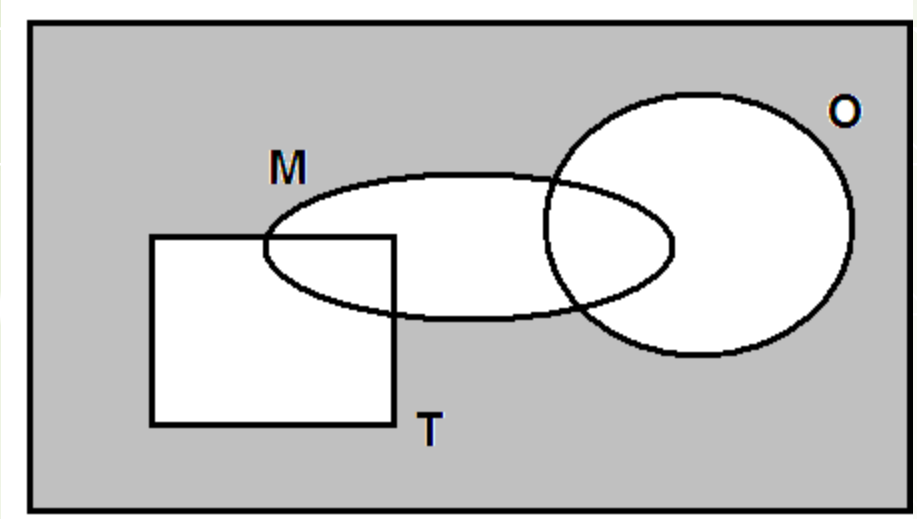
(d) Does Gerlands make T pizzas with M and O?

$$No, T \ \& \ O \text{ are Mutually exclusive}$$

PROBLEM 1.1.1

(e) Does Gerlands make R pizzas that have neither M nor O ?

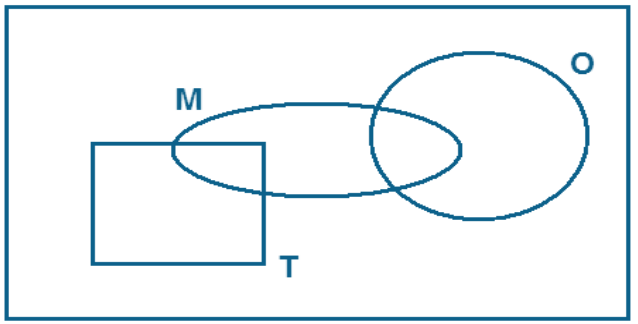
Yes, as shown in Venn diagram



PROBLEM 1.1.2

From Quiz1.1 write Gerlanda's menu in words:

- Regular with mushrooms and onions
- Tuscan with mushrooms
- Tuscan without mushrooms
- ……



1.2 Applying Set Theory to Probability

Sample Space

فضاء العينة

Sample space of an experiment is the:

- **finest grain** (all possible outcomes identified separately)
- **mutually exclusive and**
- **collectively exhaustive set.**

e.g.: Roll six-sided die one time

S={1,2,3,4,5,6}

Event

الحدث

Subset of sample space

e.g.: event E1={Roll 4 or higher}={4,5,6}

Note: event that contains all sample space is called event space

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Event

الحدث

Subset of sample space

e.g.: event E1={Roll 4 or higher}={4,5,6}

Note: event that contains all sample space is called event space

Quiz 1.2

Letter may be: voice call(v) or data call(d)
Observation a sequence of three letters .
Write the elements of the following sets.

(1) A1=First call is a voice call

A1={vvv,vvd,vdv,vdd}

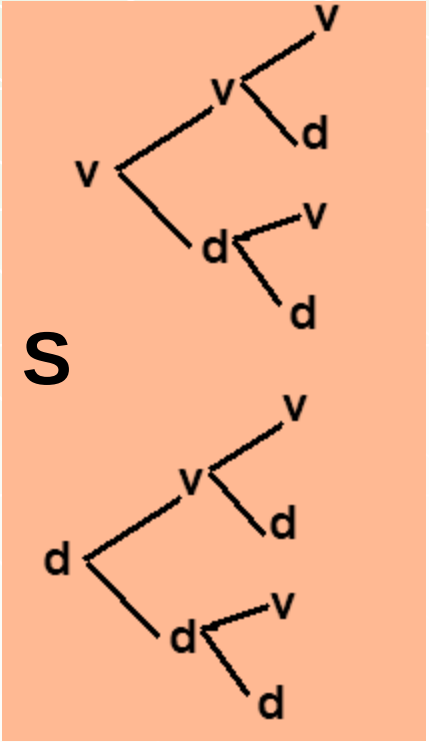
(3) A3=All call are the same

A3={vvv,ddd}

(5) B1=First call is data call

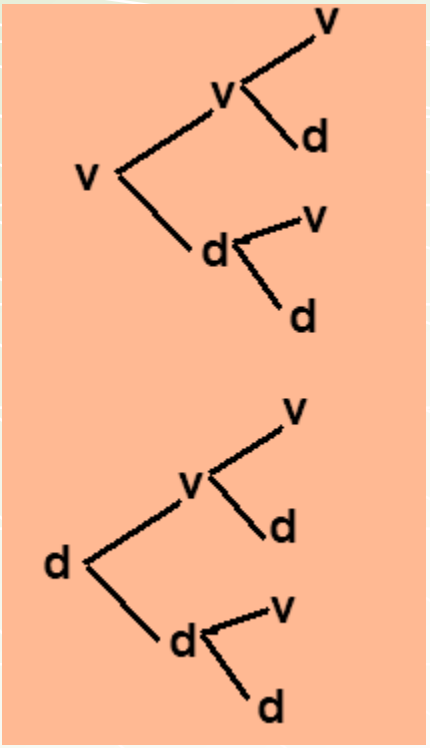
B1={dvv,dvd,ddv,ddd}

A1&B1 are Mutually exclusive and collectively exhaustive



(7) Voice and data alternate

A1={vdv,dvd}



Observation :

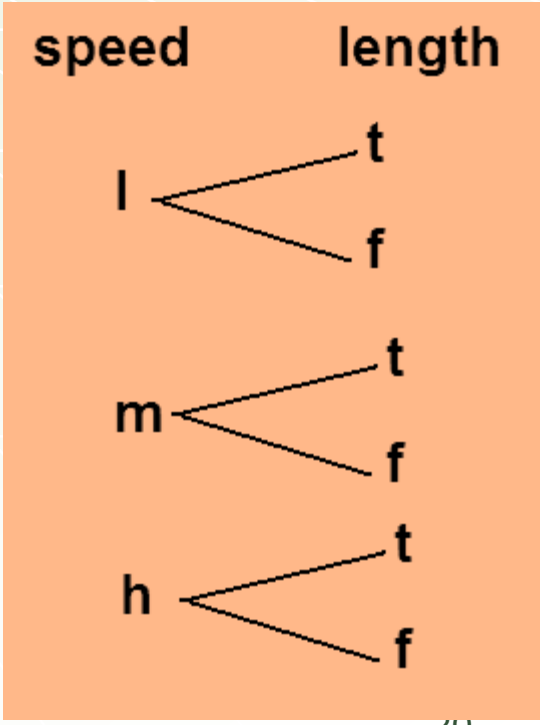
only the type(voice or data) 3 times→ $2^3 = 8$

PROBLEM 1.2.1

Experiment of monitoring a fax transmission
Speed: low(l),medium(m),high(h)
Length: short faxes 2p(t),long faxes 4p(f)

(a) What is the sample space of the experiment ?

$$S.S = \{lt, lf, mt, mf, ht, hf\}$$

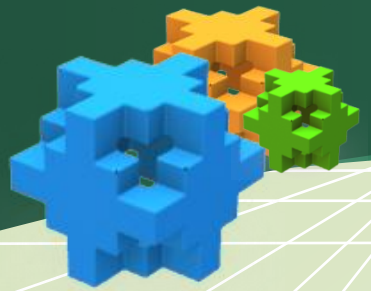


(b) Event A1={medium speed fax}

$$A1 = \{mt, mf\}$$

(c) Event A2={short fax 2p.}

$$A2 = \{lt, mt, ht\}$$



Speed (L,M,H)

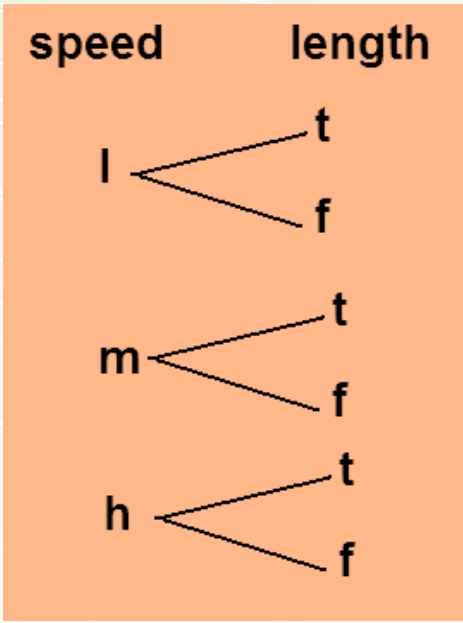
Length (Short,Long)

$$2*3=6$$

PROBLEM 1.2.1

(d) Event A3= High speed h or low speed ?

$$A3 = \{ht, hf, lt, lf\}$$



(e) Are A1,A2 and A3 Mutually exclusive

$$No, A1 \cap A2 = \{mt\} \neq \Phi$$

(f) Are A1,A2 and A3 collectively exhaustive

$$Yes, A1 \cup A2 \cup A3 = \{S\}$$

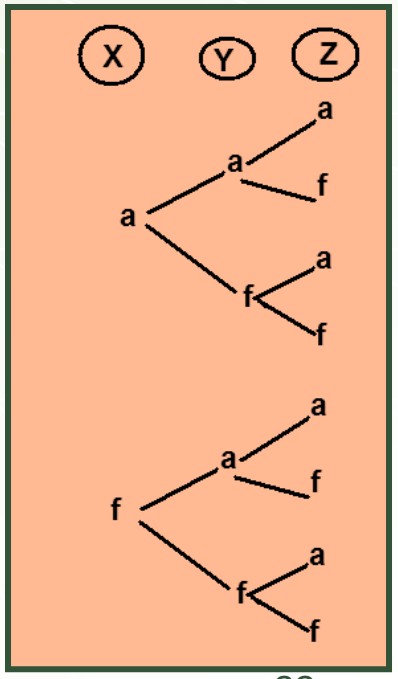
PROBLEM 1.2.2

Three machines X,Y and Z are produced integrated circuits :acceptable(a),fails(f)

Observe a sequence of three test results from the three machines ?

(a) Sample space

$$S.S = \{aaa, aaf, afa, aff, faa, faf, fffa, fff\}$$



1.3,1.4 Probability Axioms

(1) $P[A] \geq 0$

(2) $P[S]=1$

(3) $P[A \cup B]=P[A]+P[B]-P[A \cap B]$

If A&B Mutually exclusive
Then: $P[A \cup B]=P[A]+P[B]$

$$A1 \cap A2 = \Phi$$

(4) $P[\Phi]=0$

(5) $P[A^c]=1-P[A]$



(6) $P[A \cap B] = P[A,B]=P[AB]$, A and B

(7) $P[A \cap A]=P[A.A]=P[A]$

- **Equally likely Outcomes:** $$P[Si] = \frac{1}{n}$$

e.g. Roll six-sided die in which all faces are equally likely
E1={roll 4 or higher}={4,5,6}
P[E1]=3/6=1/2



Equally likely=unbiased=fair

Quiz 1.3

Test grades (T) 0-100
Score 90-100 (A) , 80-89 (B) , 70-79 (c) , 60-69 (D) ,less 60 (F)
All scores between 51-100 (S.S=50 elements)
All scores are equally likely

Find the following probability :

$$(a)\ \ P[\{S_{79}\}] = \frac{1}{50}$$

$$(d)\ \ P[T \geq 80] = \frac{21}{50} = 0.42$$

$$(c)\ \ P[A] = \frac{11}{50}$$

$$(e)\ \ P[T < 90] = \frac{39}{50} = 0.78$$

Quiz 1.3

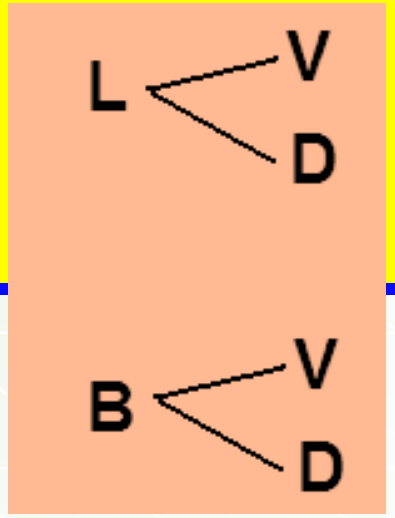
$$(f)\ P[C \text{ grade or better}] = (70 \rightarrow 100) = \frac{31}{50} = 0.62$$

$$(g)\ P[pass] = (60 \rightarrow 100) = \frac{41}{50} = 0.82$$

Quiz 1.4

Classify the call as:
Long(L) or Brief(B)
Voice(V) or Data(D)

$$S.S = \{LV, LD, BV, BD\}$$



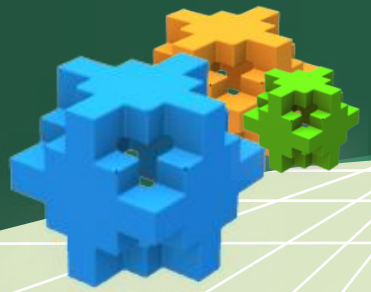
P[V]=0.7 , P[L]=0.6 , P[VL]=0.35

Find the following probabilities:

P[V]=P[LV]+P[VB] ➔P[VB]=0.7-0.35=0.35
P[L]=P[LV]+P[LD] ➔P[LD]=0.6-0.35=0.25

P[LV]+P[LD]+P[BV]+P[BD]=P[S]=1
➔ P[BD]=1-(0.25+0.35+0.35)=0.5

	V	D	
L	0.35	0.25	0.6
B	0.35	0.5	0.4
	0.7	0.3	1



	V	D	
L	0.35	0.25	0.6
B	0.35	0.5	0.4
	0.7	0.3	1

(1) $P[DL] = 0.25$

(2) $P[D \cup L] = P[D] + P[L] - P[D \cap L]$

$= 0.3 + 0.6 - 0.25 = 0.65$

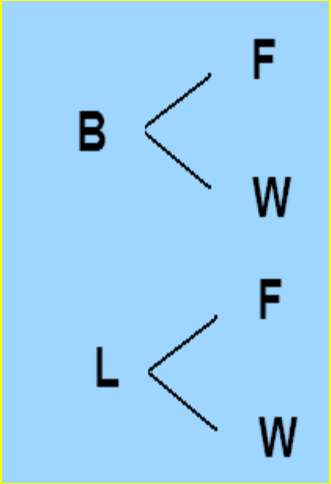
(6) $P[LB] = 0$

PROBLEM 1.3.1

Computer program are classified by:
Length of the code & Execution time

More than 150 lines → Big (B)
Less than or equal 150 lines → Little(L)

program run in less than 0.1 sec → Fast(F)
program required at least 0.1 sec → Slow(W)



P[LF]=0.5 , P[BF]=0.2 , P[BW]=0.2
What is the Sample Space(S.S) ?

$$S.S = \{BF, BW, LF, LW\}$$

	B	L
F	0.2	0.5
W	0.2	0.1

PROBLEM 1.3.1

Calculate:

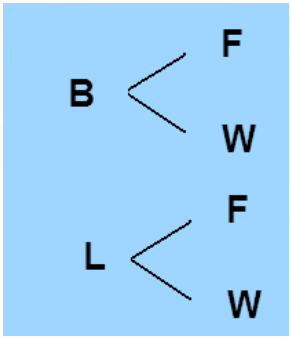
$$(a)\ P[W] = 0.2 + 0.1 = 0.3$$

$$(b)\ P[B] = 0.2 + 0.2 = 0.4$$

	B	L	
F	0.2	0.5	0.7
W	0.2	0.1	0.3
	0.4	0.6	

$$(c)\ P[W \cup B] = P[W] + P[B] - P[W \cap B] = 0.3 + 0.4 - 0.2 = 0.5$$

$$S.S = \{BF, BW, LF, LW\}$$



PROBLEM 1.3.4

Roll six-sided die once ,observe the number of dots.

What is the sample space?

$$S.S = \{1,2,3,4,5,6\}$$



What is the probability of each sample outcome ?

$$P[Si] = \frac{1}{6}$$

What is the probability of E, the event that the roll is even ?

$$E = \{2,4,6\}$$

$$P[E] = P[\{2\}] + P[\{4\}] + P[\{6\}] = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

PROBLEM 1.3.5

Quiz of 10 point
Score of students between 0 ➔ 10

- **What is the probability of A, which requires the student to get a score of 9 or more ?**

$$A = \{9,10\}$$

$$P[A] = P[\{9\}] + P[\{10\}] = \frac{1}{11} + \frac{1}{11} = \frac{2}{11}$$

- **What is the probability of F, which requires the student to get a score of less than 4?**

$$A = \{0,1,2,3\} \quad , \quad P[A] = \frac{4}{11}$$

PROBLEM 1.3.5

Quiz of 10 point
Score of students between 0 ➔ 10

- **What is the probability of A, which requires the student to get a score of 9 or more ?**

$$A = \{9,10\}$$

$$P[A] = P[\{9\}] + P[\{10\}] = \frac{1}{11} + \frac{1}{11} = \frac{2}{11}$$

- **What is the probability of F, which requires the student to get a score of less than 4?**

$$A = \{0,1,2,3\} \quad , \quad P[A] = \frac{4}{11}$$

PROBLEM 1.4.1

Mobile telephones perform *handoffs* as they move from cell to cell. During a call, a telephone either performs zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). In addition, each call is either long (L), if it lasts more than three minutes, or brief (B). The following table describes the probabilities of the possible types of calls.

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

What is the probability $P[H_0]$ that a phone makes no handoffs? What is the probability a call is brief? What is the probability a call is long or there are at least two handoffs?

(a) Find P[Ho] ?

P[Ho]=0.1+0.4=0.5

(b) Find P[B] ?

P[B]=0.4+0.1+0.1=0.6

(c) Find P[L Ụ H2] ?

=P[L]+P[H2]-P[L.H2]=0.4+0.3-0.2=0.5

	Ho	H1	H2	
L	0.1	0.1	0.2	0.4
B	0.4	0.1	0.1	0.6
	0.5	0.2	0.3	1

Thank You !



iugaza2010.blogspot.com
melasmer@gmail.com